



Exercise sheet 12

Submission: 02.07.2019

Problem 1 - Variation of Constants

(5 Points)

Let $(B_t)_{t \geq 0}$ be a Brownian motion. Let $a, \sigma : [0, \infty) \rightarrow \mathbb{R}$ be measurable and bounded functions. Calculate the solution of the SDE

(a) $dX_t = \sigma(t)X_t dB_t, \quad X_0 = 2$

Hint: By means of the Itô formula one can obtain an expression for $\int_0^t \frac{1}{X_s} dX_s$ that does not contain stochastic integrals.

(b) $dY_t = a(t)dt + \sigma(t)Y_t dB_t, \quad Y_0 = 2$

Hint: Write $Y_t = C_t X_t$ and determine a suitable C_t .

Total: 5 Points

Terms of submission:

- Solutions can be submitted in groups of at most 2 students.
- Please submit at the beginning of the lecture or until 9:50 a.m. in room 3523, Ernst-Abbe-Platz 2.